Orbital motion of test particles around a rotating neutron star

G. Urbancová[1,2], M. Urbanec[1], G. Török[1]

[1] Research Centre for Computational Physics and Data Processing, Institute of Physics, Faculty of Philosophy and Science, Silesian University in Opava, Bezručovo nám. 13, 746 01 Opava, Czech Republic, e-mail: gabriel.torok@gmail.com

[2]Astronomical Institute, Boční II 1401/2a, CZ-14131 Praha 4 - Spořilov, Czech Republic

astrocomp.physics.cz

SILESIAN UNIVERSITY IN OPAVA

EUROPEAN UNION

INSTITUTE OF PHYSICS SU OPAVA

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Abstract

Rotating neutron stars can be characterized by three main parameters - a gravitational mass M, an angular momentum J and a quadrupole moment Q. In this presentation we will demonstrate combinations of M, J, Q allowed by realistic equations of state of a dense nuclear matter. We will also discuss how these parameters affect orbital motion of test particles around rotating neutron stars. Our motivation is based on X-ray observations of binary systems with a neutron star accreting matter from its companion.

Models of rotating neutron stars

For equations of state describing nuclear matter we use SLy4 [1], UBS [2], APR [3], FPS [4], Gandolfi [5], QMC700 [6], NRAPR [7], KDE0v1 [8]. These EoSs are using various approaches to nuclear matter theory and are predicting different properties of neutron star models. We construct both non-rotating models and models rotating with the rotational frequency 400Hz.

Figure 1 shows mass-radius relations (top left) and mass as a function of the central energy density (top right) for both rotating (solid lines) and non-rotating (dashed lines) models. Each color is corresponding to one equation of state. Bottom left panel shows values of specific angular momentum j as a function of gravitational mass for stars rotating with $f_{\mathrm{rot}} = 400$Hz. Within the Hartle–Thorne approximation $j = J/M^2$ is linear function of a rotational frequency, therefore one can easily find j for another value of f_{rot} by simple linear scaling. Bottom right panel shows the value of $\tilde{q} = QM/J^2$ as a function of gravitational mass. We can see, that for neutron star massive more than $1.2M_{\odot}$ the value of $\tilde{q}$ is less than 10 and we will use this number later as maximum one in investigation of motion of particles in the field of rotating neutron stars. This will keep us in the limits of astrophysically interesting objects.

The Hartle-Thorne metric

The external Hartle-Thorne geometry can be expressed by using the three external parameters of the spacetime, gravitational mass M, dimensionless spin j and dimensionless quadrupole moment q as

$$
\begin{aligned}
g_{tt} &= +(1-2M/r)[1+j^2F_1(r)+qF_2(r)],\\
g_{rr} &= -(1-2M/r)^{-1}[1+j^2G_1(r)-qF_2(r)],\\
g_{\theta\theta} &= -r^2[1+j^2H_1(r)+qH_2(r)],\\
g_{\phi\phi} &= -r^2\sin^2\theta[1+j^2H_1(r)+qH_2(r)],\\
g_{t\phi} &= -2(M^2/r)j\sin^2\theta, \qquad (1)
\end{aligned}
$$

where F_1, F_2, G_1, H_1 and H_2 can be found in more detailed form in [9]. For $j = 0$ and $q = 0$ the Hartle-Thorne geometry reduces to the standard Schwarzschild geometry. The external Kerr geometry taken up to second order in angular momentum $a = Mj$ in the standard Boyer-Lindquist coordinates can be obtained from external Hartle-Thorne geometry, if we put $q = j^2$ and make the coordinate transformations, see [9].

Radial profiles of the orbital frequency and epicyclic frequencies

Formulas for frequencies of circular and epicyclic motion in the Hartle-Thorne geometry has been calculated in [9] and used by many authors, see e.g. [10, 11]. Here we give the relations for the orbital (Keplerian) frequency and the radial epicyclic and vertical epicyclic frequencies that are necessary for the application of the twin HF QPO models based on the geodesic quasi-circular motion and were presented in [10].

The Keplerian frequency is given by

$$\nu_K(r;M,j,q) = \frac{c^3}{2\pi GM}\frac{M^{1/2}}{r^{3/2}}\left[1 - j\frac{M^{3/2}}{r^{3/2}} + j^2E_1(r) + qE_2(r)\right]. \qquad (2)$$

The radial epicyclic frequency ν_r and the vertical epicyclic frequency ν_θ are given by

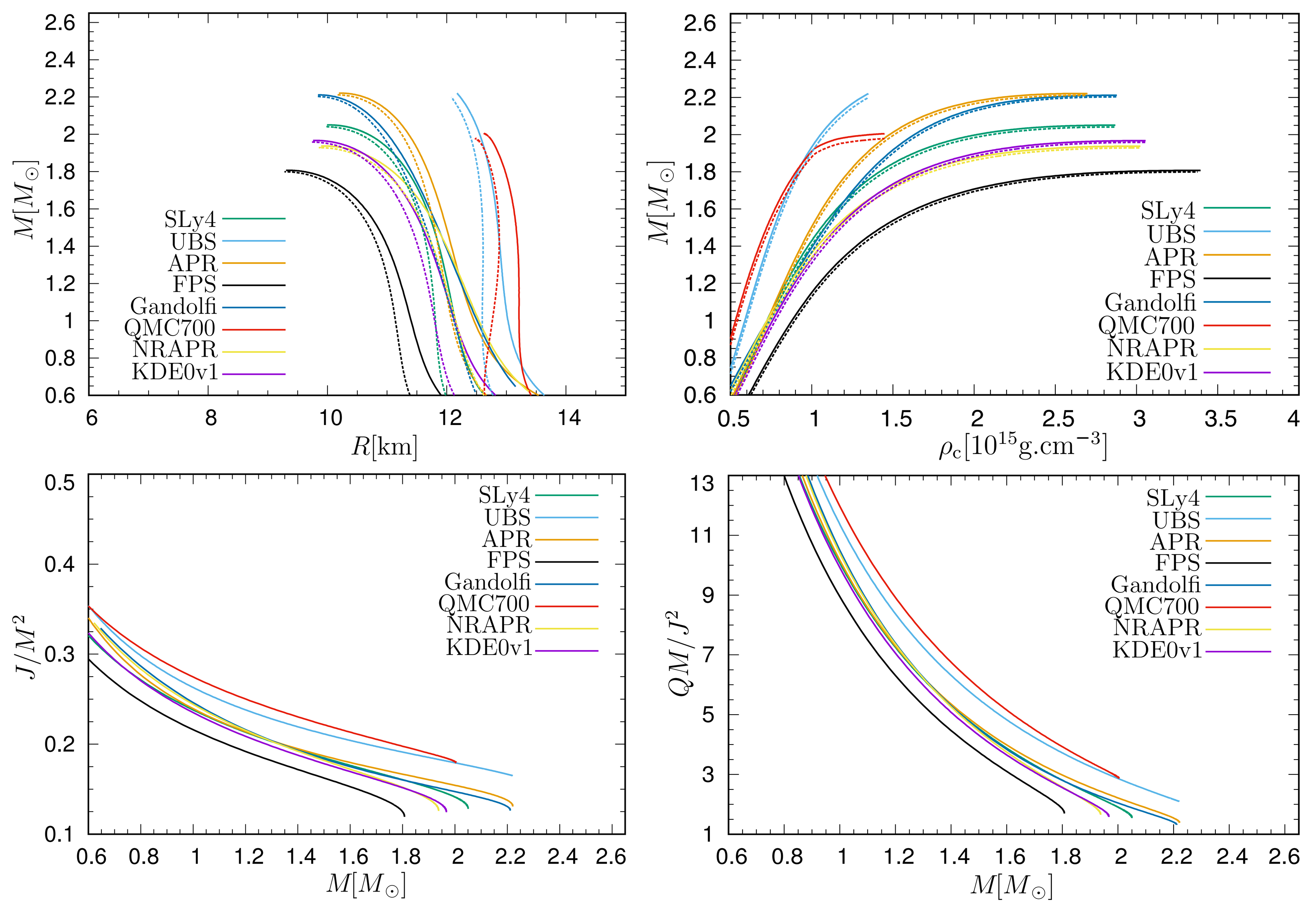
$$\nu_r^2(r;M,j,q) = \left(\frac{c^3}{2\pi GM}\right)^2\frac{(r-6M)}{r^4}[1+jF_1(r)-j^2F_2(r)-qF_3(r)], \qquad (3)$$

$$\nu_\theta^2(r;M,j,q) = \left(\frac{c^3}{2\pi GM}\right)^2\frac{M}{r^3}[1-jG_1(r)+j^2G_2(r)+qG_3(r)]. \qquad (4)$$

We first demonstrate dependence of the radial profiles of the orbital and epicyclic frequencies on the dimensionless parameters j and q. For each of the frequency profile we choose the spin parameter $j = 0.1, 0.2, 0.3, 0.5$ that covers the range of the spin parameter j when one applies Hartle-Thorne approximation to astrophysically relevant models of neutron stars or strange stars [12] and stays within the limit of slow-rotation approximation that can be written as $(j/j_{\max})^2 << 1$. It has been shown that $j_{\max} \sim 0.65 - 0.7$ for neutron stars and that $j_{\max}$ for strange (quark) stars can be larger [13]. For objects, that are less compact than compact stars the value of $j_{\max}$ can be significantly larger then values for compact stars.

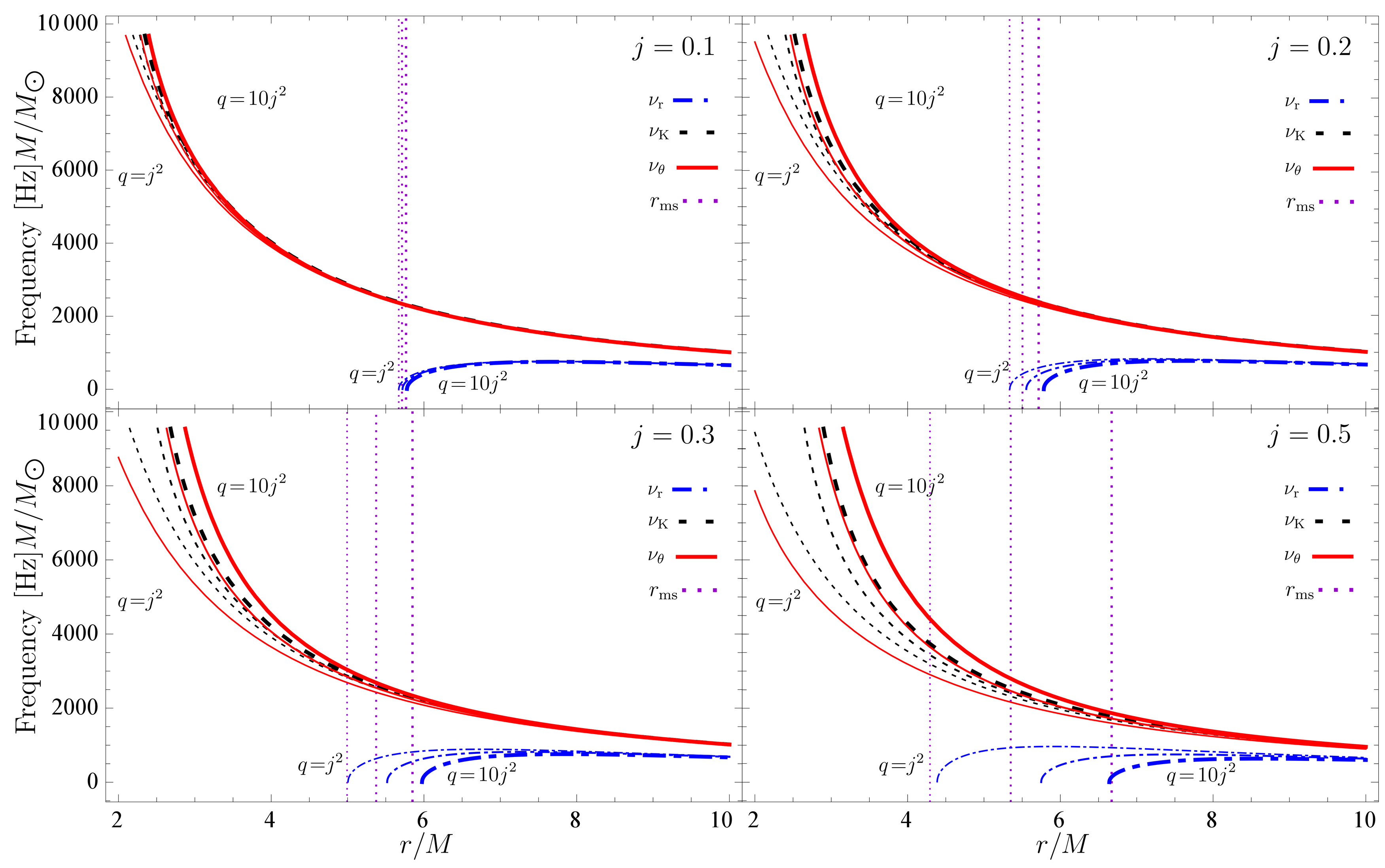
Values of specific quadrupole moment q are calculated assuming $q/j^2 = 1, 2, 3, 4, 5, 10$ for all considered values of j. Such a selection of parameters j and q covers astrophysically relevant situations and is obtained from modelling of the neutron stars as described in [12].

Figure 1



Top: Mass vs. radius (left) and mass vs. central energy density (right) for selection of equations of state, calculated for non rotating stars and stars rotating with 400Hz. Rotating stars are those having greater equatorial radius and mass and are plotted using solid lines, non-rotating models are plotted using dashed lines. Bottom: Values of $j = J/M^2$ (left) and $q = QM/J^2$ (right) vs. gravitational mass for stars rotating with the rotational frequency 400 Hz.

Figure 2



The radial profile of the orbital ν_K and the epicyclic frequencies ν_r, ν_θ for $j = 0.1, 0.2, 0.3, 0.5$. The increasing thickness of the lines is given by increasing value of $q/j^2 = 1,\ 5,\ 10$. The vertical dotted lines are calculated values of r_{ms} for each q/j^2.

Results

On Figure 1 we have shown values of parameters j, q and how they are related to mass, radius and rotational frequency of the neutron stars calculated using realistic equations of state. We used combinations of parameters obtained from realistic models of rotating neutron stars in next section and we demonstrate their impact on motion of particles in the space-times of rotating neutron stars in Hartle-Thorne approximation. On Figure 2 we can see that influence of the parameters j and q/j^2 increases with decreasing radius, and is relatively small at radii $r \sim 10M$ for all the three frequency profiles; in all the three cases the influence of the parameter q/j^2 increases with increasing value of the dimensionless spin j. For the orbital frequency the role of the parameters j and q is the smallest, and it is significantly stronger for the vertical epicyclic frequency profile that has similar character as the orbital frequency profile. The strongest influence is demonstrated in the case of the radial epicyclic frequency profile – the shift of the innermost stable circular orbit corresponding to the radius where the radial epicyclic frequency vanishes is significantly shifted even for spin $j = 0.1$, and for $j = 0.3$ the radius is shifted from $r_{\mathrm{ms}} \sim 5M$ for $q/j^2 = 1$ to $r_{\mathrm{ms}} \sim 5.8M$ for $q/j^2 = 10$, and in the extreme case of $j = 0.5$, the shift is from $r_{\mathrm{ms}} \sim 4.4M$ for $q/j^2 = 1$ to $r_{\mathrm{ms}} \sim 6.6M$ for $q/j^2 = 10$. Clearly, the role of the quadrupole moment is most significant in the behavior of the radial epicyclic motion where it affects also the value at the maxima of radial epicyclic frequency.

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